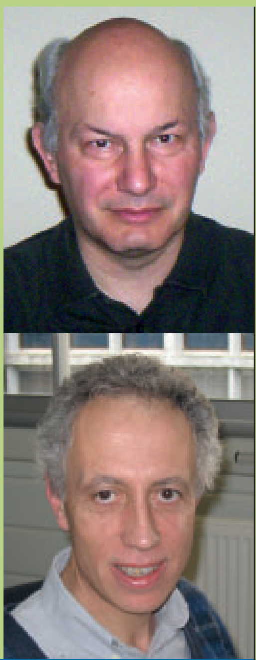
MATHEMATICS IN THE REAL WORLD



>>> Michel BOILEAU and Jean-Pierre RAYMOND professors at UPS, director and deputy-director at the IMT (joint UPS/UT1/UT2/INSA/CNRS lab).

Mathematics is often considered an abstract subject, but it is important in all scientific disciplines. With applications in industry and in everyday life.

Traditionally applied to the aerospace industry, in image treatment, cryptography and banking, mathematics is now also appearing in many other domains. These include transport (for instance, journey planning), meteorology, climate studies, risk management, data protection and music. The subject is also finding more applications in everyday life – for example, in optimising medical treatments, system security and in the important fields of energy, health and sustainable development.

New tools

The challenge is to find new tools and concepts that will help solve emerging problems linked to advances in technology and increases in computing capability. Such problems are especially pressing in the space industry – for example for calculating spacecraft trajectories or treating satellite images.

Several teams at the UPS Institute of Mathematics (IMT) are working with other researchers in the fields of biology, physics, information technology and engineering sciences. Some examples include collaborations with IMFT, INRA, IRIT, LAAS and LAPLACE. The scientists work on a variety of subjects, ranging from modelling, scientific calculations, visualisation, infinite and finite-dimension systems, algorithms and optimisation. They also work with the high-tech industry and pubic sector companies involved in cutting edge research. Some examples include Airbus, France-Telecom, CNES, DGA, ACTIS, Fluent, Dassault, CEA and IFP.

The IMT even plays a leading role in some projects. For example, in the ANR APAM (Acoustique et paroi multi-perforée) project that models thin films used in the aerospace industry in partnership with ONERA. The Hi-infinie project, based on non-differentiable optimisation techniques with ANR Guidage, ANR Controvert and FNRAE Survol, is a another example. Other joint projects in the aerospace industry include ANR CORMORED (Contrôle Optimal et Robuste par Modèles d'Ordre Réduit d'Ecoulements Décollés).

In the domain of climate studies, an IMT team is involved in two ANR projects, ADAGE, in glaciology and AMAC in flood prediction. Multi-physical models such as fluid-structure interactions are also being studied at the IMT, in collaboration with the ANR CISIFS (Contrôle et Identification dans les systèmes d'interaction fluide-structure) project.
The IMT is involved in several other modelling collaborations that will be outlined in this issue's dossier.

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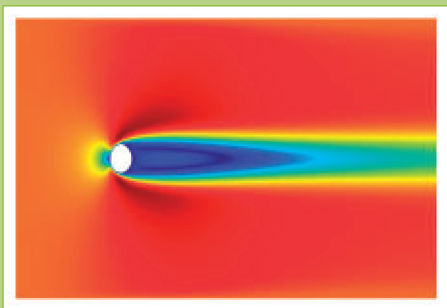
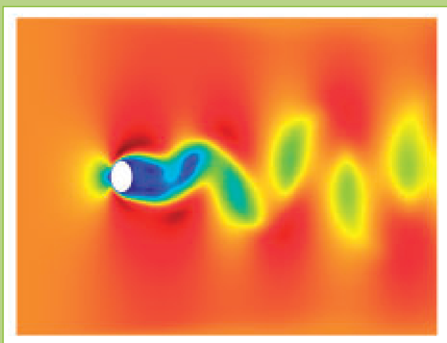
IMT: Institut de Mathématiques de Toulouse/ Toulouse Mathematics Institute
CRCA : Centre de Recherche sur la Cognition Animale/Research Center on Animal Cognition
DGA : General Delegation for Ordnance



>>> Velocity fields around a cylinder, controlled .



>>> Velocity fields around a cylinder, non-controlled.

Applying mathematics to the virtual piano, Pianoteq

Pianoteq is the very first truly modeled piano. Far beyond sampling-based digital pianos and piano software, the sound in Pianoteq is created in real time from scratch through a mathematical model that simulates an acoustic piano.

>>> Philippe GUILLAUME, professor at UPS, researcher at Institut de Mathématiques (joint UPS/CNRS lab).

The idea of modeling musical instruments is very old and has always faced great difficulties: the complexity of physical phenomena, the sensitivity of the human ear to the slightest imperfection and the difficulty of running a complex model in real-time. The latency needs to be so small that it allows the musician to feel that he is playing a real acoustic instrument. Until now, attempts to make such instruments have only confirmed that the task is not easy.

The state-of-the-art of digital pianos is based on sampling technology. Each note is a recording of how it sounds at a specific moment, without taking into account the complexity of the instrument. The huge data generated by sampling can reach 40 Gbytes for a single piano. The flow rate of data transmitted from the hard drive to the sound device is too high for current hardware capacity and one can sometimes hear crackles. Moreover, the reproduced sound lacks vividness.

Hence, creating a piano model which takes into account the interaction between hammer and strings, the interaction between strings and soundboard via the bridge and the interaction of the soundboard with air is of great interest. Based on mathematical models, Pianoteq allows parameters to be stretched as long as the model permits, resulting not only in new performance styles but also in new piano sounds. Pianoteq is thus also an innovating tool for creating music and can be useful not only for musicians but also for piano manufacturers and piano tuners for simulation and training purposes.

>>> Some parameters of Pianoteq.

Pianoteq makes excellence in piano available to all. Among Pianoteq users, composers and music professionals are certainly the most excited by our innovation. Pianoteq offers what acoustic and sampled pianos cannot offer: new opportunities for creating music and a pure piano sound that is not altered by its environment (reverberation) or by recording devices.
Pianoteq's history is strongly connected to my first job as a piano tuner. At the age of 31, I then decided to start a new life by studying mathematics at the University Paul Sabatier. This led to my PhD thesis on the parameterization of vibrating phenomena, and I never imagined that this would lead to my "third life" with Pianoteq. Thanks to these two skills and to an exceptional scientific environment in Toulouse, I succeeded in identifying important phenomena responsible for generating piano sound and proposed a model that describes the entire interaction of the soundboard, strings, bridge and air.

The contribution of Julien Pommier, doctor and engineer at the Institute of Mathematics was of great importance. He implemented real-time complex dynamic simulations, resulting in a vivid instrument.

Thanks to the French law on innovation and research (1999), INSA Toulouse, the Institute of Mathematics and the start-up MODARTT helped promote Pianoteq.

At the beginning, thanks to its huge potential, Pianoteq was considered by the press as the future of digital pianos. Today, it is seen as the state-of-the-art in this domain.

Finally, our Project KIViR (Keyboard Instruments Virtual Restoration) deals with historical keyboard instruments, which cannot be used for various reasons. Our goal is to bring them to life again.

Virtual copies are placed next to the original instrument and made available to museum visitors. Thus Pianoteq represents the future, the present, and also the past of pianos.

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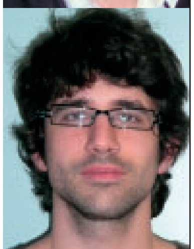
>>> Fabrice GAMBOA and Jean-Michel LOUBES, professors at UPS, researchers at the Institut de Mathématiques de Toulouse (IMT, joint UPS/CNRS lab).



>>> Philippe GOUDAL and Guillaume ALLAIN, engineers at MEDIAMOBILE.

Road traffic forecasting

Knowing exactly how long a journey will take is one the main needs of any driver. It requires a blend of good intuition and mathematical analysis.

Predicting traveling time has become a real challenge. Huge traffic jams occur everyday on road networks around cities and suburbs.

Of course, traffic models based on physics have been developed. However, they deal with too many parameters and so cannot be used to predict real-life travel time. As a matter of fact, these models provide only predictions restricted to very few points in a network.

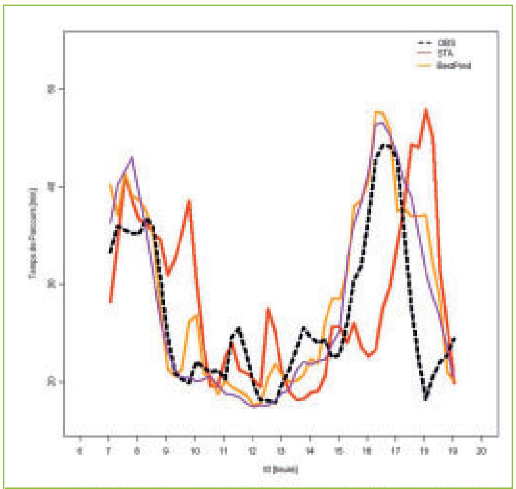
Our work aims at giving prediction time algorithms for any journey by car. It began in 2001 and received an ANVAR award in 2004 and a grant fund from ANR in 2006. It is a collaboration with the private company MEDIAMOBILE.

Modeling driver behavior

Our approach relies on the hypothesis that road traffic evolution can be summarized by two components. The first is linked to periodical behavior of users in the network. This component occurs daily and can sometimes be predicted deterministically. The second is the result of dynamic interactions in the physical flow of vehicles on the road. This component may describe the propagation of specific traffic congestion along the network, or more generally, the various reasons why traffic evolution on a specific day may locally diverge from deterministically predicted behavior.

Predictive models in each point of the network are composed of an alphabet of typical daily speed evolution profiles built upon a four-year historical database of more than 10000 points. This alphabet of typical profiles is then used to recognize the component in the mixture for the current day by using observed speeds and calendar characteristics of the day.

The model mentioned above helped us to solve several inter-independent problems: finding dissimilarities between speed curves embedding traffic characteristics, coming up with clustering methods and last but not least, defining a model identification method yielding a good trade-off between generality and accuracy.
We used complex statistical methods as non-linear regression methods and support vector machines, while proposing automatic, modular and efficient methods which also meet the industrial constraints we face.
This alphabet of typical profiles is then used to recognize the most likely evolution for the current day by using observed speeds and calendar characteristics. Our algorithm estimates short- and medium-term average vehicle speeds on an entire road network, thus providing the driver with accurate travel time predictions for his trip. This system is used on MEDIAMOBILE's website http://www.v-trafic.com/#vtactic.



>>> Observed travel times (---) and predicted by stationary method (purple) and an aggregated model (yellow)

Extending the coverage

We have also used data gathered sporadically from the analysis of in-traffic vehicles positions. This data is of interest to industry because it can be collected globally on the network, contrary to traditional data collected from counting stations made of electromagnetic loops built in the road. Accurate traffic states are reconstructed through pioneering statistical methods. These methods are based on time-space analysis of graphs and model both vehicle physics and road network geometry.

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The mathematics of complexity

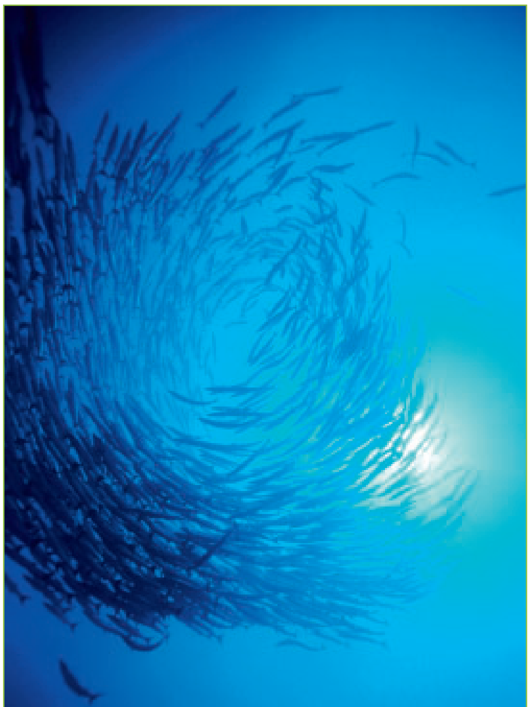
Certain systems can self-organize. Their study, in the framework of complexity mathematics, questions the very founding principles of physics.

>>> Pierre DEGOND, senior CNRS scientist at the Insitut de Mathématiques de Toulouse (IMT, joint UPS/CNRS lab) and Guy THERAULAZ, senior CNRS scientist at the Centre de Recherche sur la Cognition Animale (CRCA, UPS/CNRS).

The study of these systems, initiated by the seminal contributions of the Russian scientist Ilya Prigogine, questions some of the founding principles of physics. Indeed, the spontaneous formation of ordered structures does not correspond to the classical scheme of systems evolving towards maximal disorder, which is at the heart of the second principle of thermodynamics, and which was formalized by Boltzmann's 'entropy'.

Sahel sand dunes

Generally speaking, these systems cover a large number of autonomous mutually interacting agents. In spite of the decentralized character of the interactions, a large-scale coordination emerges. It manifests itself under the form of spatially ordered and/or temporally synchronized structures - a phenomenon referred to as 'emergence'. Examples of such systems can be found in practically all areas of science, from the science of matter (for example, granular media, network of sand dunes in the Sahel) to social sciences (economical cycles, evolution of language), and life sciences (coordination of neural oscillations, morphogenesis, collective behavior in animal societies). They now strongly inspire engineering sciences (ant-pheromone algorithms, swarm robotics).



>>> Example of emerging collective behavior: the formation of a mill structure in a school of barracuda fish.

Colossal influx of data

Since David Hilbert's address at the 1900 international mathematics congress of his 6th problem, which aimed at explaining the laws of physics, and among them, statistical physics, mathematics has provided a vast toolbox leading to an increasingly efficient modeling methodology of physical systems. This toolbox includes probability theory and partial differential equations to graph theory, geometry and combinatorics. Mathematics now plays a central part in the challenge posed by complex systems, notably because of the colossal data influx produced by contemporary experimental techniques. The search for relevant information among this huge set of data requires the use of increasingly sophisticated models that give rise to rapid validity tests of the assumptions to be made, quick selections of relevant observables, and possibly, suggestions for future experiments.

Fish schools, sheep herds

These challenges have led to the creation of a Systems Biology Institute in Toulouse, with a mathematics and computer science platform (MIBS). In this domain, the Institut de Mathématique de Toulouse (IMT) collaborates with the Centre de Recherche sur la Cognition Animale (CRCA, joint UPS-CNRS lab) on a certain number of projects: collective behavior in fish schools, gregariousness among sheep (ANR-project 'Panurge'), pedestrian and crowd dynamics (ANR-project 'Pedigree'), trail formation and nest construction by social insects. For instance, a PhD thesis on the analysis of a new model of fish displacement based on the CRCA experiments was finished last year. This model, the 'Persistent Turning Walker' supposes that fish move on circular trajectories whose radii vary according to a random process. The work undertaken during the PhD has shown that, on large temporal and spatial scales, this motion is like diffusion with a constant that shows an excellent agreement with experimental data. When a large number of fish are considered, the model describes the fish school through mean values (fish density, average velocity) and leads to very efficient numerical simulations. Thanks to this type of model, control algorithms could allow scientists to evaluate the impacts of fishing.

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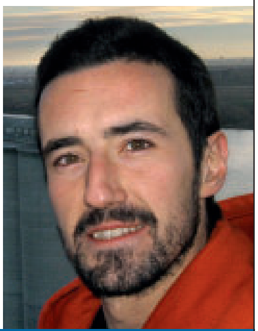
Mathematics of random networks

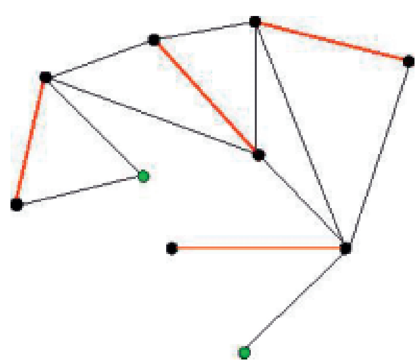


>>> Charles BORDENAVE, CNRS research scientist at l'Institut de Mathématiques de Toulouse (joint UPS/CNRS lab).

How long will it take for a pandemic to propogate? What types of roads need to be constructed to minimize traffic jams? Why do we say that everyone on the planet can be connected to another person by a network of just five people? All these questions can be answered thanks to the mathematics of networks.

A network is a set of vertices linked by edges. Networks are everywhere, in science and the everday world. For the good old road network, cities are vertices and edges are roads between them. In a social network, the vertices are individuals and there is an edge for each pair of individuals in a relationship. The network of web pages is made of web pages as vertices and hyperlinks connecting them as edges. In statistical mechanics particles may be vertices of a network and their interactions take place along the edges. In epidemiology, diseases and their treatments are spread on a network of individuals, and so on.

There are good questions to ask for each type of network. For example, in the road network, what is the length of the shortest route going through each European city ? This is the famous traveling salesperson problem and its analysis is notoriously difficult. In a social network, the distance between two individuals is defined as the shortest chain of acquaintances linking them, so what is the maximal distance between two individuals in a network ? This is the striking "small world" phenomenon and the "six degrees of separation". For the network of web pages, a popular search engine is based on return times to its initial position for a web user who would randomly click on hyperlinks. In probability, this is called a "random walk" and its properties determine the performance of the search engine. In statistical mechanics, we are interested in the energy levels of a system and its eigenstates. In epidemiology, we study the spread of propagation of an epidemic and efficient ways to stop it.



>>> The red edges are one maximal matching of the graph.

The probabilistic method

The combinatorial structure of these networks can become very complicated and it is often even unknown. To give an insightful answer to each of the above mentioned problems, it is, however, necessary to understand the geometry of the network. What all these problems have in common is that the number of vertices is large and that it is often the local structure around each vertex that plays a key role. The idea to study a random network shows up naturally at this point: instead of studying a given network, let us study a random network whose average statistical properties are believed to be close to the initial network. As the size of the network grows larger, we may hope that the answer to our problem on the random network is close to its answer in the initial network. It is even expected that the scaling limits of these random networks are the universal structures that explain the observed phenomena on large networks.

The probabilistic method also allows us to prove the existence of networks that satisfy a given property. The method then consists of drawing at random a network in a well chosen family and prove that the property is satisfied with positive probability. This was the aim of Paul Erdös in 1959 for studying random networks. Over the decades, there have been fruitful developments in many fields of mathematics, for example in number theory.

Let us conclude with an example: a matching is a subset of edges that do not share an incident vertex (see figure). Studying the maximal matchings, that is, the matchings with the maximal number of edges is an important and non-trivial issue. On random networks, with Marc Lelarge (Ecole Normale Supérieure and INRIA), we have proved the convergence of the size of maximal matching and confirmed predictions of statistical mechanics.

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Multiscale models for industry

>>> Marie-Helène VIGNAL, assistant professor at UPS and Raphaël LOUBERE, CNRS research scientist, both at the Institut de Mathématiques de Toulouse (IMT, joint UPS/CNRS lab).

Science and engineering problems often involve several space and/or time scales, and, as a consequence, are intrinsically multiscale.

Any material is constituted of atoms – that is the microscopic scale. It is also characterized by its dimensions (the macroscopic scale) that are usually much bigger than the atomic scale. Generally, the macroscopic scale corresponds to the particular scale one wishes to study. Macroscopic models that average the microscale have been used for a long time. However they are not always applicable because they might not be accurate enough and so local or global descriptions on fine microscopic scales are needed. One example of this is crack propagation. Moreover some applications can reach the limit of validity of such models - a physical macroscopic variable may cross the limit beyond which it must be considered as microscopic. In such situations, solving a microscopic model with a better accuracy and a wider range of validity is tempting. Unfortunately the difficulty and the cost of this task are very often prohibitive. Finally the sheer quantity of information not relevant to the macroscale may sometimes be too difficult to handle. In this context multiscale techniques are useful as they combine different scales. Such methods must be more efficient than a microscopic model resolution but at the same time, must produce pertinent, sufficiently accurate and easy-to-handle information.

In the following report we introduce two multiscale methods developed at IMT.
A collaborative work with DGA has adapted a Multi-scale Finite Element method (MsFEM) to simulate the transport of pollutants in an urban area. A convection-diffusion-reaction model is solved on two spatial scales; the first one is very fine (~1 meter) and allows to solve the finer details of pollutant evolution in narrow and inter-connected streets.

The second scale is a coarse 10 kilometer-scale onto which a coarse solution is computed. MsFEM consists of two steps: the precomputation of a set of elementary solutions on the coarse cells by using a fine scale resolution via a classical Finite Element (FE) method, and, a coarse scale resolution by a FE method using the previous elementary solutions as basis functions. A last phase combines the global coarse scale solution with the local elementary solutions. For the same accuracy (comparison between MsFEM and fine EF solutions) the saving in CPU time produced by MsFEM is enormous. This proves the validity of a multiscale approach in this context.

The goal of the ITER (International Thermonuclear Experimental Research) project is to generate and safely maintain fusion reactions like those produced in the Sun and other stars. A magnetic field confines an ionized gas (plasma) that is then heated for a long time in order to start an atomic fusion that subsequently generates energy. Modeling such a process is very complex as the scales, both in time and space, are very different. In the context of contracts with the CEA and Euratom we focus on the gyro-mean and low Mach number. Here, particle movement while rotating around magnetic field isolines can be averaged when the period of rotation is sufficiently small. We developed a numerical method allowing a scale transition from microscopic (if the associated scales are not too small) to the macroscopic (if the model remains valid). This approach uses the microscopic model in the whole domain, but it is solved by a CPU efficient method.

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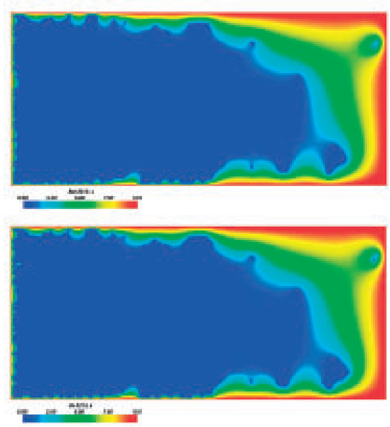
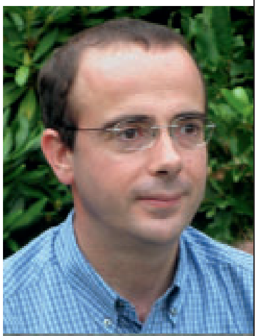


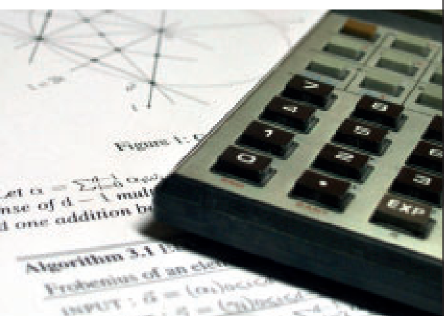
>>> Pollutant diffusion in an urban area (buildings in red, trees in green). Finite element method (left) solution in 700 seconds and Multi-scale Finite Element method in 9 $\times 10^{-2}$ second. Concentration is maximum in red, and zero in blue. Same accuracy is obtained but for a very small simulation time for MsFEM.

The art of cryptography

For a long time the domain of spies, cryptography has become a sophisticated mathematical science, with many applications in computer science.



>>> Jean-Marc COUVEIGNES, professor at UPS and researcher at the Institut de Mathématiques de Toulouse (IMT, UPS/CNRS) and Reynald LERCIER, engineer at DGA /IRMAR, Rennes.



We learn about elementary methods for adding or multiplying integers in primary school. Division comes later, together with Euclid's algorithm for computing greatest common divisors. Later on, one learns how to multiply polynomials and matrices. For every such problem, the method learnt at school is quite simple and easy to explain. It is also the most convenient for small data. Still, from an asymptotic point of view (that is for larger data), this naive method is not the best, except for addition. For example, there exist quasi-optimal algorithms for multiplying two (large) integers. Using these methods, computing the product does not require much time. One cannot hope for a faster algorithm. These fast multiplication methods rely on discrete Fourier transform and were discovered in the 1970s. They have become more widespread with the use of personal computers and computing software. As for matrix multiplication, several new methods have been discovered since 1969 that are faster than the classical multiplication of rows by columns. However, it is not known whether there exists quasi-optimal algorithms for this problem and multiplying two matrices is much slower at present. Discrete Fourier transform does not apply easily to this context because matrix multiplication is not commutative.

Computer security

Why is fast computing with big integers and big polynomials so important? On one hand, good computational methods tell us more about numbers than bad ones. On the other hand, processing digital information often requires many arithmetic operations, for example, for error correcting, enciphering, authentication, and all the functionalities of cryptography. More and more sophisticated cryptosystems are required to compete with ightthe increasing computational power available to attackers. When designing, implementing, analyzing and attacking such cryptosystems, one needs the full power of the most advanced algorithms. Unfortunately, the safest enciphering and authentication methods have a very high computational cost. This explains, for example, why transactions using smart cards can be so lengthy sometimes. There are no faster encryption techniques that can instantaneously encrypt.

Optimal cryptography

But they don't ensure quite the same level of security. Recently, the algorithmic arithmetic team at the Institut de Mathématiques de Toulouse has been looking for optimal representations of the various numbers dealt with in this context. These representations are called normal basis and they are provided by polynomial algebra and algebraic geometry. One of the results of this collaboration, with the Direction Générale pour l'Armement and the Institut de Recherche Mathématiques de Rennes, is a quasi-optimal algorithm for the generation of irreducible polynomials. This is a modest but significant achievement in the direction of a quasi-optimal cryptography. We have also investigated error correcting codes. These codes detect accidental corruption (for example, noise) of numerical signals or data, or possible dysfunction in the network. This research was a collaboration with the Institut de Mathématiques de Bordeaux and Airbus France and resulted in a patent on integrity checking in embarked networks.

All these information processing methods rely on the combinatorial properties of a limited number of very special mathematical objects that have, for a long time, attracted mathematicians' attention. This is the case of elliptic curves, modular curves, and Shimura curves, also known for their contribution to the solution of famous and long standing mathematical problems such as Fermat's last theorem or Ramanujan's conjecture. It is a remarkable fact that such elementary and speculative questions have led mathematics to ask such profound questions and to widespread applications.

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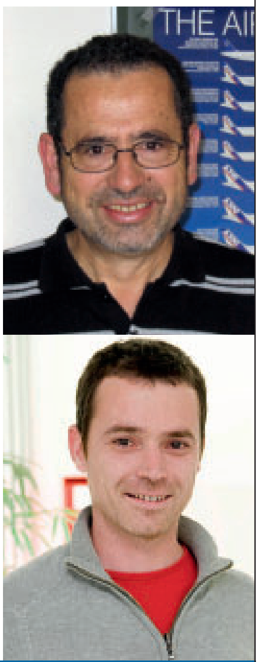
Medical Imaging: a better insight thanks to mathematics

Our approach to medical imaging is fast and accurate. It is like rubbing a pencil quickly over a sheet of paper, revealing the contours of what lies below.



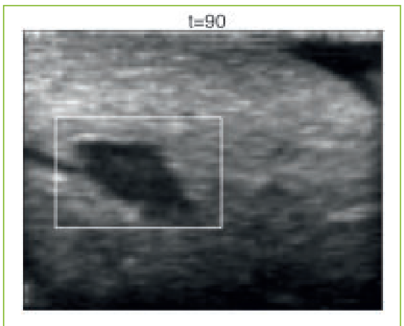
>>> Mohamed MASMOUDI, professor at UPS and Jérôme FEHRENBACH, assistant professor at UPS. Both are members of the Institut de Mathématique de Toulouse (IMT, joint UPS / CNRS lab). © C. Fresillon / CNRS.

With classical radiography, we can perform profile radiography and visualize opaque regions. If we then want to know where these regions are located, we need to take a radiograph from the front of the sample. If we want to know more about the shape of the opaque region and its size, we must take several images around the observed body. In practice, the radiographic film is replaced by collimators.

To see pathologies at their earliest stage

In medical imaging, the goal is to infer the cause (opacity) from the effect (X-ray attenuation). This is called an inverse problem. It is generally ill posed because it does not have always a solution, and when it does have a solution, it is not necessarily unique and is particularly unstable. In other words, small errors in measurement (impact on the collimator) may cause very large errors in the solution (shape of the opacity). We use regularization techniques to make the problem more stable, but the image is smoothed. This leads to the risk of eliminating important details related to a pathology, especially in the early stages of a disease.

Our team has played a pioneering role in the emergence of topological gradient imaging. If we want to decompose an image into several parts (segmentation, classification and edge detection), we just look for the characteristic function of each part. It is defined by a "1" inside the part and "0" outside. To make the problem differentiable, a classical method is to let the characteristic function take all values between 0 and 1. This increases the instability of the inverse problem. The topological gradient approach gives the variation of the energy (to minimize) when we switch the characteristic function from 1 to 0 or from 0 to 1 in a small cell (pixel). With the topological gradient, we write a new page in image processing and we plan to increase the image quality by coupling the segmentation process with inversion algorithms. We can stabilize the inversion process while using the so-called BV norm (bounded variation: sum of the norms of the gradients of the image in each pixel) regularization. This improves the quality of the image and preserves small inclusions (they have a small BV norm). This approach is not often used in medical imaging systems because of its non-differentiability. Pierre Weiss of the UPS Institute of Mathematics has shown that the non-differentiability is a source of stabilization and has proposed very efficient algorithms for solving this problem.



>>> A region of interest is monitored in real time by the algorithm developed at IMT (in-vivo data, animal testing). The goal is to follow a lesion to be treated, which is affected by respiratory movement. Collaboration with the laboratory "Applications of ultrasound therapy" Lyon.

Measurement of respiratory movements

The reconstruction process uses optimization techniques to reduce the misfit between the model and experimental measurements. Using an innovative method, we can reduce the misfit for each measurement. This method actually helps to stabilize the problem. Its application in elastography (detection of rigid inclusions), in partnership with Inserm's Unit 556, has yielded results that are far more accurate than current state-of-the-art techniques. In collaboration with the same unit, we proposed an algorithm to compensate for respiratory motion in real time during mini invasive surgical operations. These methods have been intensively used in the frame of the ANR project Addisa concerning data assimilation of images. They are also used in optical tomography in a current PhD thesis in collaboration with the Institut de Recherche en Informatique de Toulouse (IRIT). We are also working on the ANR project Mesange concerning 4D CT: our goal is to simultaneously reconstruct the cardiac vascular tree and its motion from a tomographic sequence.

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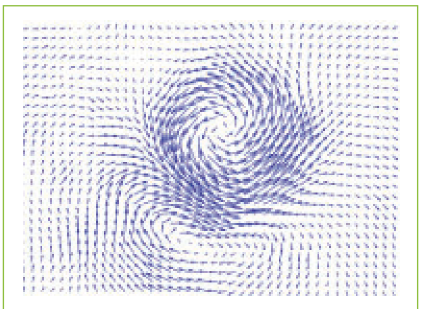

>>> Velocity field in a fluid estimated by the algorithm developed at the IMT.