

PÉRIODE D'ACCRÉDITATION : 2016 / 2021

UNIVERSITÉ PAUL SABATIER

SYLLABUS MASTER

Mention Mathématiques et applications

M2 mathématiques Research and Innovation

<http://www.fsi.univ-tlse3.fr/>
[http://departement-math.univ-tlse3.fr/
master-mention-mathematiques-et-applications-620690.kjsp](http://departement-math.univ-tlse3.fr/master-mention-mathematiques-et-applications-620690.kjsp)

2017 / 2018

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PRESENTATION

PRESENTATION OF DISCIPLINE AND SPECIALTY

DISCIPLINE MATHÉMATIQUES ET APPLICATIONS

The goal of the master "Mathematics and Applications" is to train mathematicians for careers in three possible directions :

mathematical engineering (tracks MApI3, SID, RO, SE, RI), research, innovation and development (tracks RI, RO, MApI3) and teaching (track ES).

The number of students trained in mathematics in France is much lower than the number of job offers, so that the career perspectives in mathematics are excellent.

SPECIALTY

L'objectif du parcours Research and Innovation du Master mention Mathématiques et Applications est de former des mathématiciens pouvant travailler dans les métiers de la recherche qui peut être de nature académique, théorique et/ou appliquée, ou être tournée vers l'innovation et le développement dans le secteur privé.

The goal of the specialty Research and Innovation of the Master Mathematics and Applications is to train mathematicians able to work in the research domains ranging from the academic research (both theoretical and applied) to the innovation and development in the private sector.

PRESENTATION OF THE YEAR OF M2 MATHÉMATIQUES RESEARCH AND INNOVATION

Le M2 MAT RI est structuré en deux semestres suivis d'un stage de 4 mois.

Au premier semestre les étudiants devront valider 3 cours du type "Basic course" et un "Reading Seminar".

Au deuxième semestre deux cours "Advanced Course" et le cours d'anglais complèteront la formation.

Le stage pourra être en entreprise ou dans un laboratoire de recherche et représentera une introduction à l'activité de recherche.

The M2 MAT RI is structured in two semesters followed by a 4 months stage.

During the first semester the students will validate 3 "Basic courses" and one "Reading Seminar".

During the second semester they will validate two "Advanced courses" and the english course.

The stage will be either in the industry or in a Research Laboratory, and will represent an introduction to the research activity.

CONTACTS SECTION

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TABLE SUMMARIZING THE MODULES THAT MAKE UP THE TRAINING PROGRAM

page	Code	Title of the module	ECTS	Mandatory Optional	Cours	TD	TP	Projet	Stage ne
First semester									
Choose 3 module among the following 13 modules :									
8	EIMAR3AM	BASIC COURSE 1	8	O	30				
9	EIMAR3BM	BASIC COURSE 2	8	O	30				
10	EIMAR3CM	BASIC COURSE 3	8	O	30				
11	EIMAR3DM	BASIC COURSE 4	8	O	36				
12	EIMAR3EM	BASIC COURSE 5	8	O	30				
13	EIMAR3FM	BASIC COURSE 6	8	O	30				
14	EIMAR3GM	BASIC COURSE 7	8	O	30				
15	EIMAR3HM	BASIC COURSE 8	8	O	30				
16	EIMAR3IM	SCIENTIFIC COMPUTING	8	O	30				
17	EIMAR3JM	COURS PROPOSÉ PAR UT1	8	O	30				
18	EIMAR3KM	COURS PROPOSÉ PAR UT2	8	O	30				
19	EIMAR3LM	COURS PROPOSÉ PAR L'INSA	8	O	30				
	EIMAR3MM	MACHINE LEARNING	8	O					
20	EIMAI3B1	Machine Learning			18		12		
21	EIMAI3B2	Machine Learning (projet)						25	
Choose 1 module among the following 4 modules :									
22	EIMAR3QM	READING SEMINAR 1	6	O		36			
23	EIMAR3RM	READING SEMINAR 2	6	O	24				
24	EIMAR3SM	READING SEMINAR 3	6	O		36			
25	EIMAR3TM	READING SEMINAR 4	6	O		36			
Second semester									
26	EIMAR4BM	DISSERTATION	15	O					0,1
Choose 2 module among the following 10 modules :									
27	EIMAR4CM	ADVANCED COURSE 1	6	O	24				
28	EIMAR4DM	ADVANCED COURSE 2	6	O	24				

page	Code	Title of the module	ECTS	Mandatory Optional	Cours	TD	TP	Projet	Stage ne
29	EIMAR4EM	ADVANCED COURSE 3	6	O	24				
31	EIMAR4FM	ADVANCED COURSE 4	6	O	24				
32	EIMAR4GM	ADVANCED COURSE 5	6	O	24				
33	EIMAR4HM	ADVANCED COURSE 6	6	O	24				
34	EIMAR4IM	COURS PROPOSÉ PAR L'ISAE	6	O	24				
35	EIMAR4JM	COURS PROPOSÉ PAR UT1	6	O	24				
36	EIMAR4KM	COURS PROPOSÉ PAR UT2	6	O	24				
37	EIMAR4LM	COURS PROPOSÉ PAR L'INSA	6	O	24				
Choose 1 module among the following 4 modules :									
38	EIMAR4VM	ANGLAIS	3	O		24			
39	EIMAR4WM	ALLEMAND	3	O		24			
40	EIMAR4XM	ESPAGNOL	3	O		24			
41	EIMAR4YM	FRANÇAIS GRANDS DÉBUTANTS	3	O		24			

LIST OF THE MODULES

UE	BASIC COURSE 1	8 ECTS	1st semester
EIMAR3AM	Cours : 30h		

TEACHER IN CHARGE OF THE MODULE

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LEARNING GOALS

The purpose of this course is to provide an introduction to discrete holomorphic dynamics

SUMMARY OF THE CONTENT

This course is thought of as an introduction to some aspects of complex dynamical systems with emphasis on the discrete side. The focus of the course will be on the dichotomy between local and global dynamical phenomena related to iteration of holomorphic endomorphisms in complex dimension 1, and on the relations between the local and the global aspect. If time allows, we will also deal with the higher dimensional case. Several research problems will be mentioned during the course.

REFERENCES

1. John Milnor, Dynamics in one complex variable, Annals of Mathematics Studies, 160. Princeton University Press, Princeton, NJ, 2006. viii+304 pp.
2. Xavier Buff and John H. Hubbard, Dynamics in One Complex Variable, (in preparation).

KEYWORDS

Dynamical systems, complex variables

UE	BASIC COURSE 2	8 ECTS	1st semester
EIMAR3BM	Cours : 30h		

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LEARNING GOALS

The purpose of the 2017-2018 course is to provide an introduction to K theory and vector bundles

SUMMARY OF THE CONTENT

The 2017-2018 course is to propose an introduction to vector bundles and K-Theory :

Chapter 1. Vector Bundles.

1. Basic Definitions and Constructions. Sections. Direct Sums. Inner Products. Tensor Products. Associated Fiber Bundles.
2. Classifying Vector Bundles. Pullback Bundles. Clutching Functions. The Universal Bundle. Cell Structures on Grassmannians.

Chapter 2. K-Theory.

1. The groups $K(X)$. Ring Structure. The Fundamental Product Theorem.
2. Bott Periodicity. Exact Sequences. Deducing Periodicity from the Product Theorem. Extending to a Cohomology Theory. Elementary Applications.
3. Division Algebras and Parallelizable Spheres. H-Spaces. Adams Operations. The Splitting Principle.
4. Bott Periodicity in the Real Case. (Option)
5. Vector Fields on Spheres.

Chapter 3. Characteristic Classes. (Only if time allows it)

1. Stiefel-Whitney and Chern Classes. Axioms and Constructions. Cohomology of Grassmannians.
2. Euler and Pontryagin Classes.
The Euler Class. Pontryagin Classes.

REFERENCES

A. Hatcher, K theory

J. Milnor, K theory

KEYWORDS

Vector bundle, K theory, characteristic class

UE	BASIC COURSE 3	8 ECTS	1st semester
EIMAR3CM	Cours : 30h		

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LEARNING GOALS

The purpose of this course is to provide an introduction to complex geometry

SUMMARY OF THE CONTENT

1. Holomorphic functions of several complex variables : definitions, basic properties ;
2. Laplace operator, Green-Riesz integration and representation formulae, subharmonic and plurisubharmonic functions ;
3. Basic material on currents ;
4. Notion of almost complex and complex structure ;
5. Complex analytic manifolds, smooth and holomorphic complex vector bundles, Hermitian metrics on complex vector bundles, notions of connection and curvature (generalities and the notion of Chern connection of a holomorphic Hermitian vector bundle) ;
6. Introduction to the Hodge theory of compact Kaehler manifolds (notion of Kaehler metric, examples of Kaehler and non-Kaehler manifolds, Hodge isomorphism for arbitrary compact Hermitian manifolds, Hodge decomposition and symmetry for compact Kaehler manifolds). I will do the details of what is needed from the theory of elliptic operators (including Garding's elliptic estimates) before starting the details of Hodge theory.

REFERENCES

J.-P. Demailly — *Théorie de Hodge L^2 et théorèmes d'annulation*— dans "Introduction à la théorie de Hodge", J. Bertin, J.-P. Demailly, L. Illusie, C. Peters, Panoramas et Synthèses 3, SMF (1996).

KEYWORDS

Complex geometry, holomorphic functions, Kahler manifold, Hodge theory, Laplace operator

UE	BASIC COURSE 4	8 ECTS	1st semester
EIMAR3DM	Cours : 36h		

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LEARNING GOALS

Many phenomena in physics, chemistry, biology, etc. are modeled by partial differential equations (PDE). This course provides a review of the most common PDE and their applications. Its main goal is to introduce the mathematical tools widely used in the theory of PDE.

SUMMARY OF THE CONTENT

1. Crash course : Functional analysis, distributions and Fourier transforms.
2. Sobolev spaces $H^s(\mathbb{R}^n)$ and $H^k(\Omega)$ where Ω is an open domain in \mathbb{R}^n . For future applications we will also provide the definition of Sobolev spaces $W^{k,p}(\Omega)$.
3. The Laplace, heat and wave equations in \mathbb{R}^n : modeling, representation formulae and qualitative properties.
4. Introduction to evolution equations.

PREREQUISITES

basic notions of (functional) analysis, distributions, differential equations studied in M1 and licence.

REFERENCES

H. Brézis, Analyse fonctionnelle. L. C. Evans, Partial Differential Equations. F. John, Partial Differential Equations. A. Friedman, Partial Differential Equations. G. B. Folland, Introduction to Partial Differential Equations.

KEYWORDS

Partial Differential Equation. Sobolev Spaces. Laplace equation. Heat equation. Wave equation. Fourier transform.

UE	BASIC COURSE 5	8 ECTS	1st semester
EIMAR3EM	Cours : 30h		

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LEARNING GOALS

The aim of this course is to present elementary methods used in the study of elliptic PDEs, with particular emphasis on the problems of existence, uniqueness and regularity of solutions, including the connections with the Calculus of Variations.

SUMMARY OF THE CONTENT

1. Introduction : Elliptic divergence form operators, Weak solutions.
2. Laplace's Equation : Regularity, Quantitative properties of harmonic functions (Caccioppoli's inequality, H^k estimates, Compactness....)
3. Existence of weak solutions : Lax-Milgram's Theorem, Fredholm alternative, Schauder's method, Duality method (Stampacchia's Theorem).
4. Regularity of weak solutions : DeGiorgi's theorem ($C^{0,\alpha}$ regularity), Moser's iteration technique, Schauder's estimates.
5. Maximum principle : Weak Maximum principle, Hopf's lemma, Strong Maximum principle, Harnack's inequality, Moving plane method, Symmetry of positive solutions.
6. Solutions of nonlinear elliptic PDEs : Local invertibility, Nemitskii operators, Global invertibility (Continuity method), Nonexistence results (Pohozaev's identity).
7. Calculus of variations and critical points : The Direct Method, Constrained Minimization, Euler-Lagrange equations, Morse's Deformation Theorem, Palais-Smale sequences, Mountain Pass lemma, Applications to semi-linear, elliptic PDEs.
8. Compactness : Compensated compactness (div-curl lemma), Concentrated Compactness.

PREREQUISITES

basic notions of (functional) analysis, distributions, differential equations studied in M1 and licence.

REFERENCES

Han-Lin, Elliptic partial differential equations. Evans, Partial differential equations.

Gilbarg-Trudinger, Elliptic partial differential equations of second order.

Giaquinta, Introduction to regularity theory for nonlinear elliptic systems.

KEYWORDS

elliptic operators, regularity, maximum principle, variational methods, compactness

UE	BASIC COURSE 6	8 ECTS	1st semester
EIMAR3FM	Cours : 30h		

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LEARNING GOALS

The goal of this course is to present the fundamental principles of weak convergence of probability measures in metric spaces, as well as classical functional limit theorems.

SUMMARY OF THE CONTENT

Among the main topics :

- Tight families of probability measures (theorem of Prokhorov),
- Functional limit theorems and invariance principles (Donsker's theorem),
- Gaussian measures in finite dimension (Wiener measure),
- Infinitely divisible laws, Lévy-Khinchine theorem, stable laws,
- Basics of large deviation theory in finite and infinite dimension (theorems of Cramer, Gärtner-Ellis, Schilder, Sanov,...).

The course will begin by recalling the notions of convergence for sequences of random vectors and of measures in finite dimension. It may be completed by topics chosen by the instructors (as for example : concentration of measure phenomenon, law of iterated logarithm, extreme laws, attraction domains for stable laws, applications of large deviations to spin systems....)

REFERENCES

R. M. Dudley, Real Analysis and Probability.

A. Dembo, O. Zeitouni, Large Deviations Techniques and Applications

UE	BASIC COURSE 7	8 ECTS	1st semester
EIMAR3GM	Cours : 30h		

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LEARNING GOALS

The Brownian Motion plays a fundamental role in the theory of stochastic processes. The construction of integrals with respect to this process needs the development of a specific integration theory. In the first part of the course, we will focus on this topic going from the probabilistic construction of integrals with respect to continuous martingales to the study of Stochastic Differential Equations (SDEs). Different extensions can be then studied towards integration with respect to more general processes like Levy processes or study of Malliavin Calculus

SUMMARY OF THE CONTENT

Stochastic Calculus

- (a) Brownian Motion
- (b) Martingales and Semimartingales
- (c) Stochastic Integrals
- (d) Ito Formula and Applications
- (e) Stochastic Differential Equations
- (f) Levy processes (or/and) Malliavin Calculus

REFERENCES

Chung-Williams, Introduction to stochastic integration. Ethier-Kurtz, Markov processes, characterization and convergence. Karatzas-Shreve, Brownian motion and stochastic calculus. Revuz-Yor, Continuous martingales and Brownian motion.

KEYWORDS

Brownian motion, Stochastic Differential Equation, Markov process

UE	BASIC COURSE 8	8 ECTS	1st semester
EIMAR3HM	Cours : 30h		

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LEARNING GOALS

Statistics is about the mathematical modeling of observable phenomena, using stochastic models, and about analyzing data : estimating parameters of the model and testing hypotheses. This course brings together methodological concepts, computational algorithms, a few applications and mathematical theory for asymptotic and high-dimensional statistics. The mathematical underpinning of methodology and computing has implications on exploring exciting possibilities and understanding fundamental limitations. The combination of methodology and theory builds the foundation of the course.

SUMMARY OF THE CONTENT

We present the methods and their potential for data analysis with an analysis on the underlying mathematical assumptions and properties and vice-versa, the theoretical derivations are motivated by applicability and implications to real data problems. We will present the foundations of mathematical statistics and study the properties of estimators when the size of the sample n increases. We will provide and study the statistical theory necessary for the analysis of data sets including parametric and non parametric estimation of characteristics of the data and a clustering analysis. Some illustrations on real data sets will be provided.

Outline of the lectures

1. ØDefinitions of a statistical model
2. ØTheory of Estimation and Empirical Processes.
3. ØAsymptotic Behaviour of an Estimator : rates of convergence and deviations
4. ØM-estimation in a parametric and non parametric model
5. ØTheory of data Clustering

KEYWORDS

Statistics, data clustering, Estimation

UE	SCIENTIFIC COMPUTING	8 ECTS	1st semester
EIMAR3IM	Cours : 30h		

LEARNING GOALS

This course is organized as follows : 10 academic lessons, joint with 4 numerical labs with Matlab or FreeFEM++. Each slot (either lesson or lab) lasts 2 hours. The introduction (2 slots) deals with first linear systems, second with the principles of the Finite Difference Method (FDM) for Ordinary Differential Equations (ODEs). The second part of the course (7 slots) consists in advanced material on the Finite Element Method (FEM) applied to elasticity, and on Mixed Finite Elements applied to incompressible fluid mechanics. In the last 5 slots, the last part of the course, numerical methods being used for the simulation of the propagation of electromagnetic waves are presented.

SUMMARY OF THE CONTENT

Part 1 :Introduction

1. Linear systems, condition number, decomposition of matrices for solving linear systems (EVD, SVD), least squares.
2. the Finite Difference Method (FDM) for ODEs : stability, consistency, convergence.

Part 2 :More on the Finite Element Method 3& 4. The case of coupled system of PDEs 5. Lab 1 : Finite element computations in 2D elasticity 5'. Lab 1' : Introduction to FreeFem++ 6& 7& 8. Mixed Finite Element Method : intro on the Stokes equation. Variational formulation, link with constrained optimization, inf-sup condition 9. Lab 2 : Numerical solution of the Stokes equation thanks to FreeFem++

Part 3 :Propagation models of electromagnetic waves

1. Maxwell eqns in 2D : hyperbolicity of the system, boundary conditions, energy conservation. The Yee scheme
2. FVM for Maxwell eqns in 2D. Centered fluxes scheme. L2-stability. Study of the Riemann problem. Upwind scheme

12& 13. : High order local approximation : GD methods. Variational formulation. Approximation of ordre p on triangles. Computation of the mass and stiffness matrices. Weak formulation of the boundary conditions. L2-stability of the GD method with a leap-frog scheme in the time domain 14. Lab 3 : Numerical modelling of a waveguide

REFERENCES

See the Syllabus on the M2RI's web-site for a full bibliography.

UE	COURS PROPOSÉ PAR UT1	8 ECTS	1st semester
EIMAR3JM	Cours : 30h		

UE	COURS PROPOSÉ PAR UT2	8 ECTS	1st semester
EIMAR3KM	Cours : 30h		

UE	COURS PROPOSÉ PAR L'INSA	8 ECTS	1st semester
EIMAR3LM	Cours : 30h		

UE	MACHINE LEARNING	8 ECTS	1st semester
Sous UE	Machine Learning		
EIMAI3B1	Cours : 18h , TP : 12h		

LEARNING GOALS

Ce cours a pour vocation d'introduire le concept d'apprentissage statistique et de proposer des méthodes classiques de résolution de problèmes en estimation (régression) ou classification. Une importance spécifique sera donnée aux phases de codage des méthodes statistiques.

SUMMARY OF THE CONTENT

B-A-BA de l'apprentissage statistique

1. Apprentissage Supervisé. Décomposition biais/variance. Complexité et bornes de Vapnik-Chervonenkis. Sur-apprentissage.
2. Présentation de problèmes concrets de différents domaines.
3. Training Set, Test Set. Erreur de validation croisée. Méthode bootstrap. Classification Classification CART. Etude de la méthode des K plus proches voisins. Algorithme de réseaux de neurones. Régression Estimation non paramétrique. Approximation par Fourier. Extension aux ondelettes. Régression CART. Extension à Random Forest. Régression Ridge. Critères AIC/BIC.

Apprentissage non supervisé

Présentation de situations typiques.

Méthodes de mélanges gaussien.

Classification Ascendante Hiérarchique. Algorithmes de K-means.

UE	MACHINE LEARNING	8 ECTS	1st semester
Sous UE	Machine Learning (projet)		
EIMAI3B2	Projet : 25h		

UE	READING SEMINAR 1	6 ECTS	1st semester
EIMAR3QM	TD : 36h		

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LEARNING GOALS

The purpose of this reading seminar is to let the students interested in pure mathematics learn some important and well established mathematical theories in an active way.

The students will be required to learn and expose publicly (via a weekly seminar organised in the rooms of the Institut de Mathématiques de Toulouse) different topics determined at the beginning of the year in collaboration with the organiser of the seminar.

SUMMARY OF THE CONTENT

The contents of the seminars of the students vary on a yearly basis are proposed by the organiser of the seminar and selected and distributed at the beginning of the semester.

The plan for the 2017-2018 edition is to acquire a good knowledge of the so-called Chern-Weyl theory. We will first study the basic definitions of differential manifold and of vector bundles as well as the basic operations on these objects. Then after studying the notion of connection on a vector bundle and recalling some basic facts in algebraic topology, we will define principal bundles and connections on them with the final goal of studying the relations between the curvature of these connections and the topological properties of the bundle.

PREREQUISITES

Basic knowledge of real smooth functions of many variables and of smooth manifolds. Basic topology (covering, fundamental group). The notion of Lie group.

REFERENCES

L. Nicolaescu, Lectures on the geometry of manifolds.

UE	READING SEMINAR 2	6 ECTS	1st semester
EIMAR3RM	Cours : 24h		

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LEARNING GOALS

This Reading seminar is meant for students interested in PDEs optimization and numerical analysis.

Students will explore some of the modern techniques and results in these topics under the guidance of a leading researcher of IMT.

Ce groupe de lecture est destiné aux étudiants intéressés aux EDP à l'optimization et à l'analyse numérique.

Les étudiants exploreront quelques résultats et techniques modernes de ces thématiques sous le guide d'un chercheur expert de l'IMT.

SUMMARY OF THE CONTENT

Numerical analysis of problems involving nonlinear boundary conditions(Patrick Hild)

In this reading seminar we consider several problems and their approximations by the finite element method. Most of these problems arise in solid mechanics and involve nonlinear conditions on (a part of) the boundary. The two basic problems are the one of Signorini (also called contact problem) and the Coulomb friction and both of them can be either written in a stationary or in a dynamic framework.

From a PDE point of view, only the simple static Signorini problem admits a unique solution.

For all the other problems, the mathematical analysis is not complete and the question of existence and uniqueness remains unsolved. The state of art will be recalled.

We begin with considering the simplest finite element method to approach the static Signorini problem. We study the various techniques to obtain the error bounds coming from the approximation.

We then consider the finite element discretization of the other problems and we study the properties of the finite dimensional problems, in particular the questions of existence, uniqueness, multiplicity of solutions, continuation of solutions, consistency, stability.

PREREQUISITES

Partial Differential Equations, Optimization, Numerical Analysis

REFERENCES

Ciarlet, The finite element method for elliptic problems, in Handbook of Numerical Analysis, Vol. II 1991

Haslinger, Hlaváček and Nečas, Numerical methods for unilateral problems in solid mechanics, in Handbook of Num. Anal., Vol. IV 1996

KEYWORDS

Partial Differential Equations, Optimization, Numerical Analysis

UE	READING SEMINAR 3	6 ECTS	1st semester
EIMAR3SM	TD : 36h		

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LEARNING GOALS

The aim of this reading seminar is to study some ongoing research topics in the modeling of the material sciences through a mathematical point of view.

SUMMARY OF THE CONTENT

Ginzburg-Landau vortices(Nicolas Godet, Xavier Lamy)

The Ginzburg-Landau theory is a physical model which describes superconducting materials. Ginzburg-Landau vortices are small regions where the superconductivity is destroyed.

To obtain a rigorous mathematical understanding of this phenomenon, Bethuel, Brezis and HØelein introduced and studied a simplified model of a cylindrical superconductor of planar section O . It is described by a complex map u on O which minimizes the following energy functional

$$E_\epsilon(u) = \int_O (|\text{grad } u|^2 + (|u|^2 - 1)^2 / (2\epsilon^2))$$

with fixed boundary conditions $u = g$ on the boundary of O .

The aim of this reading seminar is to study the regime where the length scale ϵ becomes very small, that is :

- \emptyset identify singular points around which the energy concentrates : the vortices
- \emptyset and obtain an asymptotic expansion of the energy depending on the positions of the vortices.

PREREQUISITES

Some familiarity with Sobolev spaces and partial differential equations will be helpful.

REFERENCES

Two articles by F. Bethuel, H. Brezis, and F. HØelein :

- Asymptotics for the minimization of a Ginzburg-Landau functional. Calc. Var. PDE, 1(2) 1993.
- Ginzburg-Landau vortices, volume 13 of Progress in Nonlinear DE and their Applications. 1994.

KEYWORDS

Calculus of variations, elliptic partial differential equations.

UE	READING SEMINAR 4	6 ECTS	1st semester
EIMAR3TM	TD : 36h		

TEACHER IN CHARGE OF THE MODULE

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LEARNING GOALS

This Reading seminar is meant for students interested in probability, statistics and their applications.

Students will explore some of the modern techniques and results in these topics under the guidance of leading researchers of IMT.

SUMMARY OF THE CONTENT

Markov processes(G. Fort and A. Joulin)

The first part of the reading seminar will be devoted to the theory of Markov chains with general state space and its application to the design and analysis of a simulation-based integration method called Markov chain Monte Carlo (MCMC) sampling. During the lectures, we will first introduce the tools for the long-time behavior analysis of the chains; and then, we will motivate the use of MCMC and introduce the classical algorithms.

For the talks, it will be proposed either to investigate further the theory of Markov chains, or to explore the use of MCMC in Bayesian Statistics or in Stochastic Algorithms for statistical Learning.

The purpose of the second part is to further complement our understanding of Markov processes, with a special emphasis on diffusion processes, the main protagonist being the Ornstein-Uhlenbeck process associated to the standard Gaussian distribution. In particular, we will study the long run behavior of these stochastic processes by means of modern probabilistic and analytical tools.

REFERENCES

- S. Meyn and R.L. Tweedie. Markov chains and Stochastic Stability
- C.P. Robert and G. Casella. Monte Carlo Statistical Methods
- D. Bakry, I. Gentil and M. Ledoux. Analysis and geometry of Markov diffusion operators.

UE	DISSERTATION	15 ECTS	2nd semester
EIMAR4BM	Stage ne : 0,1h		

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LEARNING GOALS

Permettre aux étudiants d'avoir une première expérience de recherche indépendante

SUMMARY OF THE CONTENT

Le contenu de la dissertation de stage de M2 (mémoire) est à déterminer avec l'encadrant choisi par chaque étudiant.

Une liste de propositions de stages est présentée aux étudiants au cours de l'année par les responsable du M2. Chaque stage dure au moins quatre mois et se conclut par la présentation d'un mémoire original écrit de façon indépendant par chaque étudiant.

REFERENCES

Ils sont à définir avec les encadrants des stages

UE	ADVANCED COURSE 1	6 ECTS	2nd semester
EIMAR4CM	Cours : 24h		

TEACHER IN CHARGE OF THE MODULE

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LEARNING GOALS

The purpose of this course is to provide an introduction to Hodge theory

SUMMARY OF THE CONTENT

We will introduce Kahler varieties, and the de Rham and Betti cohomology.

We will define the abstract notion of Hodge structure and we will show that for all i the Betti cohomology $H^i(X, \mathbb{C})$ of a Kahler manifold X admits a weight i Hodge structure. To conclude, we will build the Jacobian variety of a smooth complex curve.

PREREQUISITES

Notions of sheaves and cohomology of a topological manifold, of complex varieties and of complex analysis (we will recall the definition of differential form).

REFERENCES

C.Voisin, Théorie de Hodge et géométrie algébrique complexe.

Ph.Griffiths and J.Harris , Principles of algebraic geometry.

KEYWORDS

Complex variety, Kahler manifold, Hodge structure, cohomology.

UE	ADVANCED COURSE 2	6 ECTS	2nd semester
EIMAR4DM	Cours : 24h		

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LEARNING GOALS

The purpose of this course is to provide an introduction to Kähler-Einstein metrics on compact Kähler manifolds.

SUMMARY OF THE CONTENT

A fundamental problem in Kähler Geometry is to find a Kähler metric on a compact Kähler manifold with "nice properties" that reflect the topological and the differential properties of the manifold.

This has been made precise by E. Calabi in the early fifties in an ambitious program. He stated his famous conjecture and posed the problem of the existence of various kinds of "canonical metrics" on a given compact Kähler manifold.

We will prove the main theorem of S.T. Yau solving the Calabi conjecture and study the problem of the existence of Kähler-Einstein metrics on compact Kähler manifolds. These problems boil down to solving various complex Monge-Ampère equations on compact Kähler manifolds.

We will use the variational approach to solve these equations and then establish a priori estimates to study the regularity of their solutions.

This approach will need tools from Complex Differential Geometry, Pluripotential Theory and Partial Differential Equations which will be introduced in the course.

REFERENCES

J.-P. Demailly : Complex analytic and differential geometry, <https://www-fourier.ujf-grenoble.fr/~demailly/>

G. Tian : Canonical metrics in Kähler geometry, Lecture Notes ETH Zürich, Birkhäuser, 2000.

KEYWORDS

Kähler manifold, Kähler Einstein metric

UE	ADVANCED COURSE 3	6 ECTS	2nd semester
EIMAR4EM	Cours : 24h		

TEACHER IN CHARGE OF THE MODULE

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LEARNING GOALS

This course is an introduction to the controllability problems for evolution partial differential equations and more particularly for parabolic equations. We consider the following controlled evolution system :

$$\partial_t y + Ay = Bv, \quad y(0) = y_0$$

where $y(t)$ is the state at time t , y_0 the initial data, A is an elliptic operator defined on a domain Ω , $v(t)$ is the control at time t and B the control operator. The main question we will study is that of approximate (resp. null) controllability : for a given T and y_0 , does it exist a control map $t \in (0, T) \rightarrow v(t)$ such that the corresponding solution $y(t)$ of the above equation at time T is small (resp. zero) ?

The main example we shall investigate is the heat equation ($A = -\Delta$) with distributed control or boundary control.

SUMMARY OF THE CONTENT

Controllability questions for autonomous finite dimensional linear systems of ODEs : Kalman rank condition, Hautus test.

Controllability of infinite dimensional dynamical systems : Functional framework, equivalence between controllability and observability, Fattorini-Hautus test, HUM functional and applications

The heat equation :

We will study in detail this equation and introduce three methods that we will compare one with each other : the moment method in 1D, the Lebeau-Robbiano approach, the Fursikov-Imanuvilov approach. The main tools underlying those methods are the spectral properties for the operator A on the one hand, and/or Carleman estimates in another hand.

Systems of coupled parabolic equations :

We will consider here parabolic systems (with more than 1 component in the solution) that we wish to control by as few controls as possible. This topic has recently received a very careful attention in the literature and we will describe many new phenomena (some of them unexpected) that may occur in this setting.

Numerical analysis :

If there is enough time, I will also discuss issues coming from the numerical analysis of such control problems.

PREREQUISITES

basic notions of (functional) analysis, distributions, differential equations studied in M1 and licence, as well as the PDE courses of the 1st semester in M2

REFERENCES

F. Ammar-Khodja et al. Recent results on the controllability of linear coupled parabolic problems : a survey
 F. Boyer. On the penalised HUM approach and its applications to the numerical approximation of null-controls for parabolic problems.

KEYWORDS

Control theory, Partial differential equation, elliptic operator

UE	ADVANCED COURSE 4	6 ECTS	2nd semester
EIMAR4FM	Cours : 24h		

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LEARNING GOALS

In this course the design and the mathematical analysis of numerical methods for kinetic equations will be considered. Kinetic theory is a way of describing the time evolution of a system consisting of a large number of particles. This class of models is an essential tool to make the rigorous link between a microscopic description and a macroscopic description of the physical reality. Furthermore, due to the high number of dimensions and their physical properties, the construction of numerical methods is a challenge and requires a careful balance between accuracy and computational complexity.

We focus on a classical system dealing with Coulombian (or Newtonian) interactions between particles in the presence of a strong magnetic field and present the analysis of the Vlasov-Poisson system.

SUMMARY OF THE CONTENT

1 Transport equations

- \emptyset Theory of characteristics for linear equation
- Existence and uniqueness of weak and classical solutions

2 The Vlasov-Poisson system [1]

- Existence of weak solutions : solutions \emptyset a la Di Perna-Lions
- Averaging lemma : regularizing effects on transport equations - Application to the Vlasov-Maxwell system

3 Dispersion lemma

- Strichartz estimates on transport equations [3]

4 Approximation of transport equations

- Finite volume method [2]
- Stability and convergence results

PREREQUISITES

basic notions of (functional and numerical) analysis, differential equations in M1 and the PDE and Numerical Analysis courses of the 1st semester in M2

REFERENCES

F.Bouchut et al, Kinetic equations and asymptotic theory

F.Bouchut, Nonlinear stability of finite volume methods for hyperbolic conservation laws, and well-balanced schemes for sources

T.Tao, Nonlinear dispersive equations : local and global analysis

KEYWORDS

transport equation, Vlasov-Poisson system, Dispersion lemma, Approximation, Finite volume method,

UE	ADVANCED COURSE 5	6 ECTS	2nd semester
EIMAR4GM	Cours : 24h		

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LEARNING GOALS

Stochastic Algorithms(Sébastien Gadat)

This course proposes an overview of old and recent advances on the study of stochastic algorithms. Stochastic algorithms are widely used in nowadays big data problems involved in statistics, machine learning and large scale optimization (among others).

We will illustrate these methods both with a sharp description of non-asymptotic confidence bounds, and with an asymptotic point of view. The main tools are Markov chains and processes, Martingales, Deviation inequalities (for the probability and statistical side) and convex optimization with first order methods (for the optimization side).

SUMMARY OF THE CONTENT

Outline of the lectures

- 1) Reminders on the optimization of convex functions : deterministic approach
- 2) Stochastic Gradient Descent : a.s. convergence
- 3) Asymptotic analysis of SGD
- 4) Rupper Polyak Averaging principle
- 5) Optimization of non convex functions : continuous time simulated annealing

PREREQUISITES

Markov chains, Martingale theory

UE	ADVANCED COURSE 6	6 ECTS	2nd semester
EIMAR4HM	Cours : 24h		

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LEARNING GOALS

This course is an introduction to the mathematics of biological mathematics. In particular this course is focused on the development of probability in this context. Biological mathematics is a growing field of research where the Probability theory plays an essential role to understand precisely concrete situations : genetics, population evolution... Nowadays, general approaches allow to describe concrete models which involve different probabilistic tools : Markov processes, stochastic differential equations, long time behaviour...

SUMMARY OF THE CONTENT

Probability theory and applications in biological mathematics.

Mathematical models - modelisation

Markov processes - Martingales

KEYWORDS

Biological Mathematics.

UE	COURS PROPOSÉ PAR L'ISAE	6 ECTS	2 nd semester
EIMAR4IM	Cours : 24h		

UE	COURS PROPOSÉ PAR UT1	6 ECTS	2nd semester
EIMAR4JM	Cours : 24h		

UE	COURS PROPOSÉ PAR UT2	6 ECTS	2nd semester
EIMAR4KM	Cours : 24h		

UE	COURS PROPOSÉ PAR L'INSA	6 ECTS	2nd semester
EIMAR4LM	Cours : 24h		

UE	ANGLAIS	3 ECTS	2nd semester
EIMAR4VM	TD : 24h		

TEACHER IN CHARGE OF THE MODULE

CHAPLIER Claire

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LEARNING GOALS

Niveau C1 du CECRL (Cadre Européen de Certification en Langues)

Développer les compétences indispensables aux étudiant/es en vue de leur intégration dans la vie professionnelle.

Perfectionner les outils de communication permettant de s'exprimer dans le contexte international d'aujourd'hui et acquérir l'autonomie linguistique nécessaire à cette intégration

SUMMARY OF THE CONTENT

Contenu linguistique de la discipline :

Enseignement axé sur le travail de l'expression orale

Documents du domaine de spécialité pouvant faire l'objet de collaboration entre enseignants de science et enseignants de langue

Nécessité d'un parcours individualisé répondant aux attentes de chaque étudiant.

Compétences

CO - EE - EO - EE

- Savoir communiquer en anglais scientifique
- Savoir repérer les éléments constitutifs d'une communication écrite ou orale dans le domaine de spécialité
- Savoir prendre la parole en public (conférence ou réunion) dans le cadre d'un colloque, projet de recherche, projet professionnel

PREREQUISITES

N/A

REFERENCES

N/A

KEYWORDS

Projet - Repérer - Rédaction anglais scientifique - style - registre - critique - professionnel - commenter

UE	ALLEMAND	3 ECTS	2nd semester
EIMAR4WM	TD : 24h		

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UE	ESPAGNOL	3 ECTS	2nd semester
EIMAR4XM	TD : 24h		

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UE	FRANÇAIS GRANDS DÉBUTANTS	3 ECTS	2nd semester
EIMAR4YM	TD : 24h		

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GLOSSARY

GENERAL TERMS

DEPARTMENT

The departments are teaching structures within components (or faculties). They group together teachers lecturing in one or more disciplines.

MODULE

A semester is structured into modules that may be mandatory, elective (when there is a choice) or optional (extra). A module corresponds to a coherent teaching unit whose successful completion leads to the award of ECTS credits.

ECTS: EUROPEAN CREDITS TRANSFER SYSTEM

The ECTS is a common unit of measure of undergraduate and postgraduate university courses within Europe, created in 1989. Each validated module is thus assigned a certain number of ECTS (30 per teaching semester). The number of ECTS depends on the total workload (lectures, tutorials, practicals, etc.) including individual work. The ECTS system aims to facilitate student mobility as well as the recognition of degrees throughout Europe.

TERMS ASSOCIATED WITH DEGREES

Degrees have associated domains, disciplines and specialities.

DOMAIN

The domain corresponds to a set of degrees from the same scientific or professional field. Most of our degrees correspond to the domain Science, Technology and Health.

DISCIPLINE

The discipline corresponds to a branch of knowledge. Most of the time a discipline consists of several specialities.

SPECIALITY

The speciality constitutes a particular thematic orientation of a discipline chosen by a student and organised as a specific trajectory with specialised modules.

TERMS ASSOCIATED WITH TEACHING

LECTURES

Lectures given to a large group of students (for instance all students of the same year group) in lecture theatres. Apart from the presence of a large number of students, lectures are characterized by the fact they are given by a teacher who defines the structure and the teaching method. Although its content is the result of a collaboration between the teacher and the rest of the educational team, each lecture reflects the view of the teacher giving it.

TD : TUTORIALS

Tutorials are work sessions in smaller groups (from 25 to 40 students depending on the department) led by a teacher. They illustrate the lectures and allow students to explore the topics deeper.

TP : PRACTICALS

Teaching methods allowing the students to acquire hands-on experience concerning the knowledge learned during lectures and tutorials, achieved through experiments. Practical classes are composed of 16 to 20 students. Some practicals may be partially supervised or unsupervised. On the other hand, certain practicals, for safety reasons, need to be closely supervised (up to one teacher for four students).

PROJECT

A project involves putting into practice in an autonomous or semi-autonomous way knowledge acquired by the student at the university. It allows the verification of the acquisition of competences.

FIELD CLASS

Field classes are a supervised teaching method consisting of putting into practice knowledge acquired outside of the university.

INTERNSHIPS

Internships are opportunities enabling students to enrich their education with hands-on experience and to apply lessons learned in the classroom to professional settings, either in industry or in research laboratories. Internships are strongly regulated and the law requires, in particular, a formal internship convention established between the student, the hosting structure and the university.

